

11 $a = 11257^{304}$.4
 8 $4c + 2d + u$ 8 $e = \overline{cdu}$.5
 $:\mathbb{Z}/n\mathbb{Z}$ _____ •
 $\cdot \overline{a} \times \overline{b} = \overline{a \times b}$ $\overline{a} + \overline{b} = \overline{a + b}$: \mathbb{Z}^2 (a, b) : _____ -
 $\cdot \mathbb{Z}/5\mathbb{Z} = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}\}$: _____ -

$\mathbb{Z}/5\mathbb{Z}$ _____					$\mathbb{Z}/5\mathbb{Z}$ _____						
\times	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$+$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{1}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{0}$
$\overline{2}$	$\overline{0}$	$\overline{2}$	$\overline{4}$	$\overline{1}$	$\overline{3}$	$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{0}$	$\overline{1}$
$\overline{3}$	$\overline{0}$	$\overline{3}$	$\overline{1}$	$\overline{4}$	$\overline{2}$	$\overline{3}$	$\overline{3}$	$\overline{4}$	$\overline{0}$	$\overline{1}$	$\overline{2}$
$\overline{4}$	$\overline{0}$	$\overline{4}$	$\overline{3}$	$\overline{2}$	$\overline{1}$	$\overline{4}$	$\overline{4}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$

_____ : \mathbb{Z}^2 (a, b) .05 •
 $(a \equiv 0[7] \quad b \equiv 0[7]) \Leftrightarrow a^2 + b^2 \equiv 0[7]$
 $\cdot x \in \mathbb{Z} \quad x^3 \equiv 1[7] : S$.2
 $\cdot S \quad \{2, 4\} \subset S :$
 $\cdot B = 4^{1000} \quad A = 5^{2000}$.3
 $\cdot A + B \equiv 1[7] : \quad B \equiv 4[7] \quad A \equiv 4[7] :$
 $\cdot a \equiv b[n] : \mathbb{Z}^2 \quad (a, b)$.4
 $\cdot r : a! \equiv b! [n] \quad q : (a+n) \equiv b [n] \quad p : \forall x \in \mathbb{Z} : (a-x) \equiv (b-x) [n]$
 $\cdot n^3 - n \equiv 0[6] \quad n^3 - n \equiv 0[2] : \mathbb{N} \quad n$.5
 $\cdot 12 \quad 4 \quad 3 \quad P(n) = n^3 - n$

$\{\overline{0}, \overline{1}, \overline{2}, \dots, \overline{n-1}\} : n$
 $\cdot \mathbb{Z}/n\mathbb{Z} = \{\overline{0}, \overline{1}, \overline{2}, \dots, \overline{n-1}\} : \mathbb{Z}/n\mathbb{Z}$
 $\cdot n \geq 2 \quad \mathbb{N}^* \quad n \quad \mathbb{Z}^4 \quad (a, b, c, d)$.05 -
 $\cdot \begin{cases} a \equiv b [n] \\ c \equiv d [n] \end{cases} \Rightarrow \begin{cases} a+c \equiv b+d [n] \\ ac \equiv bd [n] \end{cases}$
 $\cdot \begin{cases} a \equiv b [n] \\ c \equiv d [n] \end{cases} \Rightarrow \begin{cases} \forall (u, v) \in \mathbb{Z}^2 : ua + vc \equiv ub + vd [n] \\ \forall p \in \mathbb{N} : a^p \equiv b^p [n] \end{cases}$
 $(25 \quad 11 \quad 9 \quad 5 \quad 3 \quad 2) : ______ \cdot$
 $\cdot a_n \times 10^n + a_{n-1} \times 10^{n-1} + \dots + a_1 \times 10 + a_0$ a
 $\cdot a = \overline{a_n a_{n-1} \dots a_0} : (\forall k \in \{0, 1, 2, \dots, n\} : a_k \in \{0, 1, 2, \dots, 9\})$
 $\cdot a_0 \quad (5 \quad) \quad 2 \quad a$
 $\cdot (5 \quad) \quad 2$
 $\cdot a_n + a_{n-1} + \dots + a_0 : (9 \quad) \quad 3 \quad a$
 $\cdot (9 \quad) \quad 3$
 $\cdot \overline{a_1 a_0} \quad (25 \quad) \quad 4 \quad a$
 $\cdot (\overline{a_1 a_0} \equiv 0[25] \quad) \quad \overline{a_1 a_0} \equiv 0[4] :$
 $\cdot d = a_0 - a_1 + a_2 - a_3 + \dots + (-1)^n a_n : 11 \quad a$
 $\cdot 11$
 $\cdot 11 \quad a = \overline{37568x} \quad x$.1
 $\cdot 11 \quad 3 \quad b = \overline{28x75y} \quad y \quad x$.2
 $\cdot 11 \quad 9 \quad 8 \quad c = \overline{13xy45z} \quad z \quad y \quad x$.3

$$\begin{array}{r} a \quad b \quad a > b \quad b \quad a \\ \cdot b \quad a \\ \cdot b = 4116 \quad a = 33810 \end{array} \quad : \underline{\hspace{2cm}} -$$

	$q_1 = 8$	$q_2 = 4$	$q_3 = 1$	$q_4 = 2$
$a = 33810$	$b = 4116$	$r_1 = 882$	$r_2 = 588$	$r_3 = 294$
$r_1 = 882$	$r_2 = 588$	$r_3 = 294$	$r_4 = 0$	

$\cdot 33810 \wedge 4116 = 294 :$

:06 -

$5617 \wedge 813 \quad 4847 \wedge 5633 \quad .1$

$\cdot \forall n \in \mathbb{N}^* : (5n^3 - n) \wedge (n+2) = (n+2) \wedge 38 : - .2$

$(5n^3 - n) \wedge (n+2)$

$\mathbb{N}^* \quad n \quad -$

$(2) : (5n^3 - n) \wedge (n+2) = 19 \quad (1) : (n+2) \mid (5n^3 - n)$

:07 -

$\cdot a \wedge b = \delta \Leftrightarrow \exists (u, v) \in \mathbb{Z}^2 / \delta = au + bv : \mathbb{Z}^* \quad b \quad a$

$a \wedge b = \delta \quad b \quad a \quad : \underline{\hspace{2cm}} -$

$\cdot \delta = au + bv : \mathbb{Z}^2 \quad (u, v)$

$b \quad a \quad b = 35 \quad a = 120 \quad : \underline{\hspace{2cm}} -$

	$q_1 = 3$	$q_2 = 2$	$q_3 = 3$
$a = 120$	$b = 35$	$r_1 = 15$	$r_2 = 5$
$r_1 = 15$	$r_2 = 5$	$r_3 = 0$	

$\cdot 120 \wedge 35 = 5 :$

$15 = 120 - 3 \times 35 : \quad 120 = 3 \times 35 + 15 :$

$5 = 35 - 2 \times 15 = 35 - 2 \times (120 - 3 \times 35) = -2 \times 120 + 7 \times 35$

$\cdot 5 = -2 \times 120 + 7 \times 35 :$

: -III

:1

$b \quad a \quad D_a \cap D_b \quad b \quad a$

$d = a \wedge b \Leftrightarrow d \mid a \quad d \mid b \quad (\forall k \in \mathbb{Z} / k \mid a \quad k \mid b : k \leq d) \quad \bullet$

$D_{30} = \{-30, -15, -6, -5, -2, -1, 1, 2, 5, 6, 15, 30\} : \underline{\hspace{2cm}} -$

$D_{54} = \{-54, -27, -18, -9, -6, -3, -2, -1, 1, 2, 3, 6, 9, 18, 27, 54\}$

$\cdot 30 \wedge 54 = 6 : \quad D_{30} \cap D_{54} = \{-6, -2, -1, 1, 2, 6\} :$

:2

$\cdot a \wedge b = b \wedge a = |a| \wedge |b| = a \wedge |b| = |a| \wedge |b| : \mathbb{Z}^* \quad b \quad a \quad \bullet$

$\cdot a \wedge b = |b| \Leftrightarrow b \mid a \quad a \wedge 1 = 1 \quad a \wedge 0 = |a| : \mathbb{Z} \quad a \quad \bullet$

$\cdot (ab) \wedge (ac) = |a|(b \wedge c) : \mathbb{Z}^3 \quad (a, b, c) \quad \bullet$

$\left(\frac{a}{d}\right) \wedge \left(\frac{b}{d}\right) = \frac{(a \wedge b)}{|d|} : \quad b \quad a \quad d \quad \bullet$

$\cdot \left(\frac{a}{\delta}\right) \wedge \left(\frac{b}{\delta}\right) = 1 : \quad a \wedge b = \delta$

$\cdot a \wedge b = (a - bq) \wedge b : \mathbb{Z}^* \quad q \quad b \quad a \quad \bullet$

:3

:06 -

$r \quad a \quad b \quad a > b \quad b \quad a$

$\cdot a \wedge b = b \wedge r : \quad b \quad a$

$(a \quad b \quad a > b) \quad b \quad a$

$\cdot a \wedge b = b \wedge r_1 \quad 0 < r_1 < b \quad a = bq_1 + r_1 : \quad b \quad a$

$\cdot b \wedge r_1 = r_1 \wedge r_2 \quad 0 \leq r_2 < r_1 \quad b = r_1q_2 + r_2 : \quad b$

$a \wedge b = r_1 : \quad r_1 \wedge 0 = r_1 \quad r_2 = 0$

$r_k = 0 \quad r_2 \neq 0$

$\cdot a \wedge b = r_{k-1} :$

$ax + by = c$ -V

:01 -

$(E): 5x + 3y = 1$: \mathbb{Z}^2

$(E) \quad (-1, 2) \quad 5 \times (-1) + 3 \times 2 = 1 :$

$5(x + 1) + 3(y - 2) = 0 :$ $\begin{cases} 5x + 3y = 1 \\ 5 \times (-1) + 3 \times 2 = 1 \end{cases} :$

$5(x + 1) = -3(y - 2) :$

$(3 \wedge 5 = 1 \quad 3 | 5(x + 1) :) \quad 3 | (x + 1) :$

$k \in \mathbb{Z} \quad y = 2 - 5k \quad x = -1 + 3k :$

$S = \{(-1 + 3k, 2 - 5k) / k \in \mathbb{Z}\} :$

:09 -

$\mathbb{Z}^2 \quad ax + by = 1$

$b \quad a$

$(x_0, y_0) \quad S = \{(x_0 + kb, y_0 - ka) / k \in \mathbb{Z}\}$

:09 -

$(E): 437x - 241y = 1$: \mathbb{Z}^2

:02 -

$(E): 26x + 65y = 13$: \mathbb{Z}^2

$\forall (x, y) \in \mathbb{Z}^2 : 26x + 65y = 13 \Leftrightarrow 2x + 5y = 1 :$

$2x + 5y = 1 \quad (-2, 1) \quad 2 \times (-2) + 5 \times 1 = 1 :$

$S = \{(-2 + 5k, 1 - 2k) / k \in \mathbb{Z}\} : (E)$

: -

$ax + by = a \wedge b \Leftrightarrow a'x + b'y = 1 : \quad b \quad a$

$b = (a \wedge b)b' \quad a = (a \wedge b)a' :$

$S = \{(x_0 + kb', y_0 - ka') / k \in \mathbb{Z}\} : \quad a' \wedge b' = 1$

$a'x + b'y = 1 : \quad (x_0, y_0)$

$ax + by = c$ -IV

:01 .1

$a \wedge b = 1 :$

$b \quad a$

:02 .2

(théorème de Bezout)

$\mathbb{Z}^2 \quad (u, v)$

$b \quad a$

$au + bv = 1$

:07 -

$\mathbb{N}^* \quad n$.1

$(3n - 2) \wedge (5n - 3) = 1 \quad (2n + 1) \wedge (9n + 4) = 1 \quad n \wedge (n + 1) = 1$

$\frac{n^2 - 7n + 15}{n - 3}$

$n \geq 4 \quad \mathbb{N} \quad n$.2

$\frac{n^2 - 7n + 15}{n - 3}$

$n \geq 4 \quad \mathbb{N} \quad n$.3

(théorème de Gauss)

$c \quad b \quad a$

$a | c : \quad a \wedge b = 1 \quad a | bc$

: -

n

$(ab \equiv ac [n] \quad a \wedge n = 1) \Rightarrow b \equiv c [n] : \quad \mathbb{Z}^* \quad c \quad b \quad a$

:08 -

$a > b \quad a \wedge b = 1 : \quad \mathbb{N}^* \quad b \quad a$

$(a + b) \wedge (a - b) = 2 \quad (a + b) \wedge (a - b) = 1$

:07 -

$(a \wedge b = 1 \quad a \wedge c = 1) \Leftrightarrow a \wedge (bc) = 1 : \quad \mathbb{Z}^3 \quad (a, b, c)$

$(a \wedge b = 1 \quad c \wedge d = 1) \Leftrightarrow (ac) \wedge (bd) = 1 : \quad \mathbb{Z}^4 \quad (a, b, c, d)$

$a \wedge b = 1 \Rightarrow \forall (n, m) \in \mathbb{N}^2 : a^n \wedge b^m = 1 :$

:08 -

$(a | c \quad b | c \quad a \wedge b = 1) \Rightarrow (ab) | c : \quad \mathbb{Z}^3 \quad (a, b, c)$

_____ -VI
:_____ .1

$b \ a$
ppcm(a,b) $a \vee b$

$a \vee a = |a| \ a \vee 1 = |a| : \mathbb{Z}^* \ a \ b | a \Leftrightarrow a \vee b = |a|$
:_____ .2

$a \vee b = b \vee a = |a| \vee |b| = |a \vee b| : \mathbb{Z}^{*2} \ (a,b) \bullet$
 $(ab) \vee (ac) = |a|(b \vee c) : \mathbb{Z}^{*3} \ (a,b,c) \bullet$

$\left(\frac{a}{d}\right) \vee \left(\frac{b}{d}\right) = \frac{(a \vee b)}{|d|} : \ b \ a \ d \bullet$

$(a \vee b) \times (a \wedge b) = |ab| \ a \wedge b = 1 \Leftrightarrow a \vee b = |ab| : \mathbb{Z}^{*2} \ (a,b) \bullet$
:_____ :11 -

$\mathbb{N}^* \ n \ n \vee (n+2) \ .1$

$\forall (a,b) \in \mathbb{N}^2 : a \wedge b = 1 \Leftrightarrow (a+b) \wedge (ab) = 1 : \ .2$

$\forall (a,b) \in \mathbb{N}^2 : a \wedge b = (a+b) \wedge (a \vee b)$
: $\mathbb{N}^2 \ .3$

(1): $\begin{cases} a+b = 68 \\ a \vee b = 240 \end{cases}$

:12 -

$B = 9n - 5 \ A = 3n + 4 : \mathbb{N}^* \ n \ .1$

$\mathbb{N}^* \ n \ A \wedge B \ -$
(2): $\begin{cases} A \wedge B = 17 \\ A \vee B = 884 \end{cases} : \mathbb{N}^* \ n \ -$

. 1980 - .2

(3): $(a \vee b)^2 - 5(a \wedge b)^2 = 1980 : \mathbb{N}^2 \ -$

(4): $\begin{cases} x \vee y = 210(x \wedge y) \\ y - x = x \wedge y \end{cases} : \mathbb{N}^2 \ .3$

$c \in \mathbb{Z} \ (a,b) \in \mathbb{Z}^{*2} \ (E): ax + by = c$
:_____ :03 -

(E): $27x - 39y = 12 : \mathbb{Z}^2$

(E) $\Leftrightarrow 9x - 13y = 4 : \ 3|12 \ 27 \wedge 39 = 3 :$

$9x - 13y = 1 : \ (3,2) \ 9 \times 3 - 13 \times 2 = 1 :$

$(12,8) : \ 9x - 13y = 4$

$(E) \Leftrightarrow 9(x - 12) = 13(y - 8)$

(E) $k \in \mathbb{Z} \ y = 8 + 9k \ x = 12 + 13k$

$S = \{(12 + 13k, 8 + 9k) / k \in \mathbb{Z}\}$

$c | (a \wedge b) \ \mathbb{Z}^2 \ (E): ax + by = c$

$c = (a \wedge b)c' \ b = (a \wedge b)b' \ a = (a \wedge b)a' :$

$a' \wedge b' = 1 \ (E) \Leftrightarrow a'x + b'y = c'$

$a'x + b'y = c' : \ a'x + b'y = 1 : \ (x_0, y_0)$

(E) $\Leftrightarrow a'(x - c'x_0) = b'(c'y_0 - y) : \ (c'x_0, c'y_0)$

$k \in \mathbb{Z} \ y = c'y_0 - ka' \ x = c'x_0 + kb' :$

$S = \{(c'x_0 + kb', c'y_0 - ka') / k \in \mathbb{Z}\} :$

:10 -

: $\mathbb{Z}^2 \ .1$

(3): $7x - 9y = 1 \ (2): 3x - 5y = 13 \ (1): 3x - 4y = 10$

(6): $26x + 15y = 4 \ (5): 4x + 3y = 5 \ (4): 39x - 45y = 6$

. 7 64 52 $n \ .2$

$\begin{cases} 2x + 3y = 78 \\ -6 \leq x \leq 21 \\ -5 \leq y \leq 14 \end{cases} : \mathbb{Z}^2 \ .3$

$n^p \equiv n^{p+4} \pmod{p}$ $\forall n \in \mathbb{N}; \forall p \in \mathbb{N}^* : n^{p+4} \equiv n^p \pmod{p}$: .2

$\forall a \in \mathbb{Z} : a^{561} \equiv a \pmod{561}$: $\forall a \in \mathbb{Z} : \begin{cases} a^{561} \equiv a \pmod{3} \\ a^{561} \equiv a \pmod{11} \\ a^{561} \equiv a \pmod{17} \end{cases}$: .5

$p \geq 3$ $(p-1)^2 \equiv 1 \pmod{p}$: .i

$\forall x \in \mathbb{Z} : x^2 \equiv 1 \pmod{p} \Leftrightarrow (x \equiv 1 \pmod{p} \vee x \equiv -1 \pmod{p})$: .ii

$\forall x \in \{1, 2, 3, \dots, p-1\}; \exists u \in \{1, 2, 3, \dots, p-1\} / ux \equiv 1 \pmod{p}$: .iii

$(p-1)! \equiv (p-1) \pmod{p}$: .iv

$(p-1)! \equiv (p-1) \pmod{p} \Rightarrow p$: .v

(théorème de wilson) $(p-1)! \equiv -1 \pmod{p} : p \geq 3$: .-VIII

$n = \varepsilon p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k} : |n| \geq 2$ n

$p_k \dots p_2 p_1 (\alpha_1, \alpha_2, \dots, \alpha_k) \in \mathbb{N}^{*k}$ $k \in \mathbb{N}^*$ $\varepsilon = \text{sg}(n) = \pm 1$

$p_k \dots p_2 p_1$ n n .

$\varepsilon = \pm 1$ $d = \varepsilon p_1^{\beta_1} p_2^{\beta_2} \dots p_k^{\beta_k} : n$ d .

$\{1, 2, \dots, k\}$ i $0 \leq \beta_i \leq \alpha_i$

$\varepsilon = \pm 1$ $m = \varepsilon p_1^{\gamma_1} p_2^{\gamma_2} \dots p_r^{\gamma_r}$ n m .

$\{k+1, k+2, \dots, r\}$ i $\gamma_i \geq 0$ $\{1, 2, \dots, k\}$ i $\alpha_i \leq \gamma_i$

$p \in \mathbb{N}$ $p \geq 2$ \mathbb{N} p : .-VII

29 23 19 17 13 11 7 5 3 2 : .1

2 $n \geq 2$: .-13

$(2^n - 1) \Rightarrow (n)$ n : .2

$p \wedge q = 1 : p \neq q$ q p : .

$p \wedge a = 1$ a p \mathbb{Z} a : .

$\forall k \in \{1, 2, 3, \dots, p-1\} : p \wedge k = 1$: .3

p n : .10

$p^2 \leq n$ p n : .11

p $a_n \dots a_2 a_1$: .12

$p | (a_1 a_2 \dots a_n) \Leftrightarrow \exists k \in \{1, 2, \dots, n\} / p | a_k$: .13

P : .4

$\forall k \in \{1, 2, \dots, p-1\} : p | C_p^k$: .1

$\forall n \in \mathbb{N} : n^p \equiv n \pmod{p}$: .2

$\forall n \in \mathbb{N} : n \wedge p = 1 \Rightarrow n^{p-1} \equiv 1 \pmod{p}$: .3

(le petit théorème de Fermat) $\forall a \in \mathbb{Z} : a^p \equiv a \pmod{p}$: .p

(Fermat) $a^{p-1} \equiv 1 \pmod{p} : p \wedge a = 1$: .-15

$\forall n \in \mathbb{N}; \forall p \in \mathbb{N}^* : n^{p+4} \equiv n^p \pmod{p}$: $\forall n \in \mathbb{N} : n^5 \equiv n \pmod{5}$: .1

$$\begin{aligned}
 & : b \in \mathbb{N}^* - \{1\} \quad b \quad .2 \\
 & . b \geq 2 \quad b \quad : \quad - \\
 a = a_k b^k + a_{k-1} b^{k-1} + \dots + a_1 b + a_0 : & \quad k \quad \mathbb{N} \quad a \\
 . a_k \neq 0 \quad \forall i \in \{0, 1, \dots, k\} : 0 \leq a_i \leq b-1 : & \\
 . b \quad a \quad a = \overline{a_k a_{k-1} \dots a_1 a_0}_{(b)} : & \\
 & : \quad - \\
 & : \quad \mathbf{b-1} \quad \mathbf{0} \\
 .9 \quad \dots \quad \mathbf{1} \quad \mathbf{0} : (&) \\
 \beta \quad \alpha \quad \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \alpha, \beta\} : & \quad 12 \\
 & \mathbf{11} \quad \mathbf{10} \\
 a = 10 \times 12^4 + 2 \times 12^3 + 12^2 + 11 \times 12 + 7 & \quad \overline{\alpha 21 \beta 7}_{(12)} : \\
 & : \mathbf{18} \quad \bullet \\
 : \quad x.y \quad y-x \quad x+y : & \quad \mathbf{4} \quad \mathbf{5} \quad \mathbf{1} \quad \mathbf{2} \\
 . y = \overline{1341}_{(5)} \quad x = \overline{232}_{(5)} & \\
 21 = 4 \times 5 + 1 \quad 85 = 4 \times 21 + 1 \quad 342 = 4 \times 85 + 2 : & \quad .1 \\
 . 1 = 4 \times 0 + 1 \quad 5 = 4 \times 1 + 1 & \\
 342 = 4(4 \times 21 + 1) + 2 & : \\
 = 4^2(4 \times 5 + 1) + 4 + 2 & \\
 = 4^3(4 \times 1 + 1) + 4^2 + 4 + 2 & \\
 = 4^4 + 4^3 + 4^2 + 4 + 2 & \\
 342 = \overline{11112}_{(4)} : & \\
 \mathbf{4} \quad \mathbf{4} \quad \mathbf{342} & \\
 . \mathbf{4} \quad \mathbf{342} & \\
 y = b^3 + 3b^2 + 4b + 1 \quad x = 2b^2 + 3b + 2 : & \quad b = 5 \quad .2 \\
 x + y = \overline{2123}_{(5)} : & \quad x + y = 2b^3 + b^2 + 2b + 3 : \\
 y - x = \overline{1104}_{(5)} : & \quad y - x = b^3 + b^2 + 4 \\
 . x.y = \overline{433212}_{(5)} : & \quad x.y = 4b^5 + 3b^4 + 3b^3 + 2b^2 + b + 2
 \end{aligned}$$

$$\begin{aligned}
 & : b \quad a \quad \{p_1, p_2, \dots, p_k\} \quad b \quad a \quad : \mathbf{14} \quad - \\
 \{1, 2, \dots, k\} \quad i \quad \beta_i \geq 0 \quad \alpha_i \geq 0 : & \quad b = p_1^{\beta_1} p_2^{\beta_2} \dots p_k^{\beta_k} \quad a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k} \\
 . \forall i \in \{1, 2, \dots, k\} : \begin{cases} \gamma_i = \min(\alpha_i, \beta_i) \\ \delta_i = \max(\alpha_i, \beta_i) \end{cases} : & \quad \begin{cases} a \wedge b = p_1^{\gamma_1} p_2^{\gamma_2} \dots p_k^{\gamma_k} \\ a \vee b = p_1^{\delta_1} p_2^{\delta_2} \dots p_k^{\delta_k} \end{cases} : \\
 & : \mathbf{17} \quad - \\
 . r \times 10^m \in \mathbb{Z} \quad \mathbb{N} \quad m \quad r & \quad r \quad .1 \\
 . p \wedge q = 1 \quad q \in \mathbb{N}^* \quad p \in \mathbb{Z} \quad r = \frac{p}{q} & \\
 . r \Leftrightarrow \exists (\alpha, \beta) \in \mathbb{N}^2 / r = 2^\alpha \times 5^\beta : & \\
 . n! \quad p \quad n \geq 2 \quad \mathbb{N} \quad n & \quad .2 \\
 14! \quad n! \quad p & \quad - \\
 100! & \quad - \\
 p_r \quad \dots \quad p_1 \quad \alpha_i \geq 1 \quad n = \prod_{i=1}^r p_i^{\alpha_i} : & \quad n \geq 2 \quad \mathbb{N} \quad n \quad .3 \\
 . S(n) = \prod_{i=1}^r \frac{p_i^{\alpha_i+1} - 1}{p_i - 1} : & \quad n \quad r \quad - \\
 : \quad n \wedge m = 1 \quad m \geq 2 \quad n \geq 2 \quad \mathbb{N} \quad m \quad n & \quad - \\
 S(m.n) = S(m).S(n) & \\
 & : \quad -\mathbf{VIV} \\
 & : \quad \mathbf{1} \\
 9 \quad \dots \quad 2 \quad 1 \quad 0 : & \quad \bullet \\
 . (&) \quad \mathbf{10} \quad \bullet \\
 8 \times 10^3 + 2 \times 10^2 + 5 \times 10 + 6 : & \quad \mathbf{8 \ 256} \quad - \\
 . \mathbf{4 \ 300 \ 970} : & \quad 4 \times 10^6 + 3 \times 10^5 + 9 \times 10^2 + 7 \times 10 \\
 \mathbf{1 \ 0} : & \quad \bullet \\
 & \quad \mathbf{2} \\
 29 = \overline{11101}_{(2)} : & \quad 29 = 1 \times 2^4 + 1 \times 2^3 + 1 \times 2 + 1 : \quad \mathbf{29} \quad - \\
 & \quad \mathbf{29}
 \end{aligned}$$