

•  $(E_1): z^8 \bar{z}^{-3} - 1 = 0 : \mathbb{C}$  -**(10)**  
 $(E_2): z^2 - 2z + 1 + \cos 2\alpha - i \sin \alpha = 0 :$  -**(11)**  
 •  $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$

**تمرين 02:** •  
 :  $(O, \bar{e}_1, \bar{e}_2)$  (P)  
 $(\Gamma_2) = \left\{ M(z) \in (P) / \left( \frac{z-i}{z+i} \right)^2 = i \right\}$   $(\Gamma_1) = \{ M(z) \in (P) / |z+3i| = 2|z| \}$   
 •  $(\Gamma) = \left\{ M(z) \in (P) / \left( \frac{z-i}{z+i} \right)^2 = i \left| \frac{2z}{z+3i} \right| \right\}$   
 •  $r$   $\Omega$   $(\Gamma_1)$  -**(1)**  
 $(D_2)$   $(D_1)$   $(\Gamma_2) = ((D_1) \cup (D_2)) - \{A\} :$  -**(2)**  
 •  $(\Gamma)$   $(\Gamma) = (\Gamma_1) \cap (\Gamma_2) :$  -**(3)**

**تمرين 03:** •  
 $(E): z^2 - mz + m(1+i) - 2i = 0 : \mathbb{C}$   
 •  $m \in \mathbb{C}$   
 •  $(E)$   $S_m$  -**(1)**  
 •  $(E)$   $m$  -**(2)**

**تمرين 04:** •  
 :  $\mathbb{C}$   $\mathbb{R}^* \times \mathbb{R}$   $(\lambda, \theta)$   
 •  $(E'): z^2 - 2(\lambda \cos \theta + i \sin \theta)z + \lambda^2 - 1 = 0$   
 •  $\mathbb{R}^* \times \mathbb{R}$   $(\lambda, \theta)$   $z_2$   $z_1$   $(E')$  -**(1)**  
 •  $(E')$   $z_2$   $z_1$   $\theta$   $\lambda$  -**(2)**

**تمرين 01:** •  
 •  $z_0^{15}$   $z_0^3$   $z_0^2$   $z_0 = \frac{5+3\sqrt{3}i}{1-2\sqrt{3}i}$  -**(1)**  
 :  $\mathbb{C}$  -**(2)**  
 $(E_3): iz^2 - 2\bar{z} + 2 - i = 0$   $(E_2): z^2 - 4\bar{z} + 4 = 0$   $(E_1): \bar{z} + |z| = 6 + 2i$   
 $z_1^2$  •  $z_1 = \sqrt{2-\sqrt{3}} - i\sqrt{2+\sqrt{3}}$  -**(3)**  
 •  $z_1$   
 $(P)$  •  $z' = \frac{z+2}{z-2i} : \mathbb{C} - \{2i\}$   $z$  -**(4)**  
 :  
 $(\Gamma_2) = \{ M(z) \in (P) / z' \in \mathbb{R} \}$   $(\Gamma_1) = \{ M(z) \in (P) / |z'| = 1 \}$   
 $(\Gamma_4) = \{ M(z) \in (P) / \arg(z') \equiv \pi [2\pi] \}$   $(\Gamma_3) = \{ M(z) \in (P) / z' \in i\mathbb{R}^* \}$   
 :  $(\Sigma_1)$   $(P)$  -**(5)**  
 •  $(\Sigma_1) = \{ M(z) \in (P) / z + \bar{z} + z\bar{z} = 0 \}$   
 $\mathbb{C}$   $z_c$  •  $B(\sqrt{3})$   $A(i)$   $(P)$  -**(6)**  
 •  $ABC$   
 $(P)$   $M(z)$   $(\Sigma_2)$  -**(7)**  
 •  $P(-3i)$   $N(i\bar{z})$   $M(z)$   
 •  $1+z_1z_2 \neq 0$   $|z_1|=|z_2|=1 : \mathbb{C}$   $z_2$   $z_1$  -**(8)**  
 •  $\frac{z_1+z_2}{1+z_1z_2} \in \mathbb{R} :$   
 $\forall (z_1, z_2) \in \mathbb{C}^2 : |z_1 - z_2| = |1 - \bar{z}_1 z_2| \Leftrightarrow (|z_1|=1 \text{ أو } |z_2|=1) :$  -**(9)**  
 •  $\frac{z_1 - \bar{z}_1 z_2}{1 - z_2} \in \mathbb{R} \Leftrightarrow (z_1 \in \mathbb{R} \text{ أو } |z_2|=1) : z_2 \neq 1$

**تمرین 09**

$(S_n)_{n \in \mathbb{N}^*}$   $z = \cos \frac{\pi}{n} + i \sin \frac{\pi}{n}$  :  $\mathbb{N}^* \quad n$

$S_n = \sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \dots + \sin \frac{(n-1)\pi}{n}$  :

$\forall n \in \mathbb{N}^* : 1 + z + z^2 + \dots + z^{n-1} = i \cdot \frac{\sin \frac{\pi}{n}}{\left(1 - \cos \frac{\pi}{n}\right)}$  : - (1)

$\left(\frac{S_n}{n}\right)_{n \in \mathbb{N}^*}$   $\forall n \in \mathbb{N}^* : S_n = \frac{1}{\tan\left(\frac{\pi}{2n}\right)}$  : - (2)

**تمرین 10**

$z_2 = [r, \theta]$   $z_1 = [r, \alpha]$  :  $\mathbb{R}_+^* \quad r \quad \mathbb{R}^2 \quad (\alpha, \theta)$  - (1)

$z_2 \quad z_1$   
 $q \quad p \quad (E) : z^2 - 2pz + q = 0$  : - (2)

$\frac{p^2}{q} \quad (E)$

$(0 < \frac{p^2}{q} \leq 1) \cdot ]0,1]$

$z_4 = [r', \alpha]$   $z_3 = [r, \alpha]$  :  $\mathbb{R}_+^{*2} \quad (r, r') \quad \mathbb{R} \quad \alpha$  - (3)

$z_4 \quad z_3$   
 $q \quad p$  - (4)

$(E) : z^2 - 2pz + q = 0$

$(0 < \frac{q}{p^2} \leq 1) \cdot ]0,1]$

$\frac{q}{p^2}$

**تمرین 05**

$\omega = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$  :  $\omega$

$(2) : \omega^5 - 1 = 0$   $(1) : \cos \frac{2\pi}{5} = \frac{1}{2} \left(\omega + \frac{1}{\omega}\right)$  : - (1)

$(3) : 1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$

$\cos \frac{2\pi}{5}$   $(4) : X^2 + X - 1 = 0$  :  $X = \omega + \frac{1}{\omega}$  - (2)

**تمرین 06**

$z_0 = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$   $T = z_0^3 + z_0^5 + z_0^6$   $S = z_0 + z_0^2 + z_0^4$  :

$\text{Im}(S) > 0$   $T \quad S$  - (1)

$T \quad S \quad ST \quad S+T$  - (2)

**تمرین 07**

$(E_2) : z^4 + 1 = 0$   $(E_1) : z^4 - 1 = 0$  :  $\mathbb{C}$  - (1)

$(E) : 1 + z + z^2 + \dots + z^6 + z^7 = 0$  :

$(E') : \left(\frac{2z+1}{z-i}\right)^4 = 1$  :  $\mathbb{C}$  - (2)

$(C) \quad (P)$

**تمرین 08**

$\mathbb{N}^* \quad n \quad \mathbb{R} - \{2k\pi/k \in \mathbb{Z}\} \quad x$

$Y_n = \sin x + \sin 2x + \dots + \sin nx$   $X_n = 1 + \cos x + \cos 2x + \dots + \cos nx$

$X_n + iY_n = \frac{e^{i(n+1)x} - 1}{e^{ix} - 1}$  :  $\mathbb{N}^* \quad n$

$Y_n = \sin(nx/2) \cdot \frac{\sin((n+1)x/2)}{\sin(x/2)}$   $X_n = \cos(nx/2) \cdot \frac{\sin((n+1)x/2)}{\sin(x/2)}$



$z'' = \frac{z}{z-i} \quad z' = z - i$			
$(O, \vec{e}_1, \vec{e}_2)$	(P)		-3
$N(z'')$	$M(z')$		
$N(z'')$	$M(z')$	$(\Gamma_2)$	$(\Gamma_1)$
$]0, \pi[$	$\varphi$		
<b>تمرين 19</b>			
$\mathbb{C}$	$]-\frac{\pi}{2}, \frac{\pi}{2}[$	$\alpha$	
$(E): (1+iz)^3(1-i \tan \alpha) - (1-iz)^3(1+i \tan \alpha)$			
$z \in \mathbb{R}$	$ z+i  =  z-i $	(E)	$z$
	$e^{i\alpha}$	$\frac{1+i \tan \alpha}{1-i \tan \alpha}$	-2
	$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$	$z = \tan \varphi$	$\mathbb{R} \quad z$
	(E)	$\varphi$	(E')
	(E)		
<b>تمرين 20</b>			
	$\mathbb{C}$		-1
$(E): z^3 - 2(\sqrt{3}+i)z^2 + 4(1+\sqrt{3}i)z - 8i = 0$			
	(E)		أ-
	(E)		ب-
	$\mathbb{R}$	$\theta$	-2
$(E_\theta): z^3 - 2e^{i\theta}(\sqrt{3}+i)z^2 + 4e^{2i\theta}(1+\sqrt{3}i)z - 8ie^{3i\theta} = 0$			
(E)	$ze^{-i\theta}$	إذا و فقط إذا كان	أ-

$m$	$R$	$\Omega$	$O$	$AMN$	ج-
	$A$		$AMN$		
<b>تمرين 16</b>					
		$z^2 - 2iz - 2 = 0$	$\mathbb{C}$		-1
	(E)	$z^2 - 2ze^{i\theta} + e^{2i\theta} - 1 = 0$	$]0, \pi[$	$\theta$	-2
	(E)	$\mathbb{C}$			
$(O, \vec{e}_1, \vec{e}_2)$			(P)		-3
$z_3 = -1 + e^{i\theta}$	$z_2 = 2e^{i\theta}$	$z_1 = 1 + e^{i\theta}$	$C(z_3)$	$B(z_2)$	$A(z_1)$
	$OABC$		$z_3$	$z_1$	أ-
	$OABC$	$]0, \pi[$	$\theta$		ب-
<b>تمرين 17</b>					
	$(E_\theta): z^3 + 2(1 - \cos \theta)z^2 + (1 - 4 \cos \theta)z + 2 = 0$		$\mathbb{C}$		
		$0 < \theta < \pi$			
	$\theta$	$z_0$	$(E_\theta)$		-1
	$\text{Im}(z_1) < 0$	$z_2 \quad z_1$	$(E_\theta)$		-2
$(O, \vec{e}_1, \vec{e}_2)$			(P)		-3
	$C(z_2)$	$B(z_1)$	$A(z_0)$		
	$ABCD$		$D$	$z_D$	أ-
	$ABCD$	$]0, \pi[$	$\theta$		ب-
<b>تمرين 18</b>					
	$z$	$[-\pi, \pi]$	$\varphi$		-1
	$z = \frac{1}{2}[\sin \varphi + i(1 - \cos \varphi)]$				
		$]0, \pi[$	$\varphi$		-2

**تمرين 23**

- (P)  $(O, \bar{e}_1, \bar{e}_2)$  (P)  $M(z)$   $T$
- (P)  $M'(z')$   $z \neq -i$  (P)  $M(z)$   $T$
- $z' = \frac{iz+3}{z+i}$
- (P) - (1)
- $(\Gamma) = \{M(z) \in (P) / z' \in \mathbb{R}\}$   $(\Delta) = \{M(z) \in (P) / |z'| = 1\}$
- $(\Gamma_-) = \{M(z) \in (P) / z' \in \mathbb{R}_-^*\}$   $(\Gamma_+) = \{M(z) \in (P) / z' \in \mathbb{R}_+^*\}$
- $\mathbb{C} - \{-i\}$   $z$   $(z' - i)(z + i) = 4$  : - (2)
- (P)  $B$   $A$   $(\overline{e_1, BM'}) = -(\overline{e_1, AM})[2\pi]$   $AM \cdot BM' = 4$
- $z_B = i$   $z_A = -i$
- $A$  (C) (P) - (3)
- $T$   $R$
- (P) - (4)
- $(C_-) = \{M(z) \in (C) / \text{Im}(z) \leq -1\}$   $(C_+) = \{M(z) \in (C) / \text{Im}(z) \geq -1\}$
- $\mathbb{C}$  - (5)
- (E) :  $(z-a)^n - (z-b)^n = 0$
- $n \geq 2$   $n$   $a \neq b$   $b$   $a$
- (E) - (ب)
- $z_k = \frac{a+b}{2} + i \frac{a-b}{2} \cotan\left(\frac{k\pi}{n}\right)$  : (E) - (ب)
- $k \in \{1, 2, 3, \dots, n-1\}$  :

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ب- ثم إستنتج حلول كل معادلة من المعادلتين  $(E_\theta)$   $(E_\pi)$

$(E_\pi)$   $(E)$  (P) -ج

(C)

**تمرين 21**

- $\sqrt{2}$   $m$  (E) :  $mz^2 - 2z + \bar{m} = 0$  :  $\mathbb{C}$  - (1)
- (E)  $\alpha \in \mathbb{R}$   $m = \sqrt{2}e^{i\alpha}$  - (2)
- $z'' = e^{-i(\frac{\pi}{4}+\alpha)}$   $z' = e^{i(\frac{\pi}{4}-\alpha)}$
- (P) - (3)
- $(O, \bar{e}_1, \bar{e}_2)$  (P)
- $M''(z'')$   $M'(z')$   $M(z'+z'')$
- $OM''MM''$   $\overline{OM'} \perp \overline{OM''}$  :

**تمرين 22**

- (P) (P)
- $C(10+2i)$   $B(4-i)$   $A(2+3i)$
- $M'(z')$   $M(z)$  (P) (P)  $T'$   $T$
- $z'' = (1+j)z$   $z' = jz$  :  $M''(z'')$
- $T'$   $T$   $B$   $ABC$  - (1)
- (P)  $ABC$  (C) - (2)
- $(C'') = T'(C)$   $(C') = T(C)$  :
- ( $\Delta$ ) ( $\Delta$ ) - (3)
- $c$   $b$   $a$   $a(z+\bar{z}) + bi(z-\bar{z}) + c = 0$
- $(\Delta'') = T'(\Delta)$   $(\Delta') = T(\Delta)$  :

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