

**:01** •

$$\cdot (u_n)_{n \in \mathbb{N}} \quad u_0 \quad r \quad - (1)$$

$$\cdot u_7 + u_8 + \dots + u_{12} = -129 \quad u_0 + u_1 + \dots + u_5 = -3$$

$$\cdot (x_n) \quad - (2)$$

$$\cdot (4): x_n = \frac{2n^{\frac{3}{2}} + 3^n}{n^2 + 4^n} \quad (3): x_n = \frac{1+2+3+\dots+n}{n^2} \quad (2): x_n = \frac{4^n - \pi^n}{3^n - \pi^n} \quad (1): x_n = \left( \frac{1-\sqrt{2}}{\sqrt{2}-\sqrt{3}} \right)^{2n}$$

$$\cdot \left( \frac{n!}{2^n} \right)_{n \in \mathbb{N}} \quad \forall n \in \mathbb{N} / n \geq 7 : n! > 3^n \quad - (3)$$

**:02** •

$$\cdot \forall n \in \mathbb{N}^* : u_n = \frac{C^{2n}}{4^n} \quad (u_n)_{n \in \mathbb{N}^*}$$

$$\cdot (u_n)_{n \in \mathbb{N}^*} \quad \forall n \in \mathbb{N}^* : u_{n+1} = \frac{2n+1}{2n+2} u_n \quad - (1)$$

$$\cdot (u_n)_{n \in \mathbb{N}^*} \quad - (2)$$

$$\cdot (u_n)_{n \in \mathbb{N}^*} \quad \forall n \in \mathbb{N}^* : u_n < \frac{1}{\sqrt{2n+1}} \quad - (3)$$

**:03** •

$$\cdot \forall n \in \mathbb{N} : x_{n+2} = x_{n+1} - \frac{1}{4} x_n \quad x_1 = 1 \quad x_0 = -1 \quad (x_n)_{n \in \mathbb{N}}$$

$$\cdot v_n = x_{n+1} - \frac{1}{2} x_n \quad u_n = 2^n x_n \quad \mathbb{N} \quad n$$

$$\cdot n \quad v_n \quad v_0 \quad (v_n)_{n \in \mathbb{N}} \quad - (1)$$

$$\cdot u_0 \quad (u_n)_{n \in \mathbb{N}} \quad - (2)$$

$$\cdot \forall n \in \mathbb{N} / n \geq 2 : \left( \frac{3}{2} \right)^n \geq n \quad n \quad x_n \quad u_n \quad - (3)$$

$$\cdot (x_n)_{n \in \mathbb{N}} \quad \lim_{n \rightarrow +\infty} \frac{n}{2^n} \quad - (4)$$

**:04** •

$$\forall n \in \mathbb{N} : u_{n+2} = 5u_{n+1} - 4u_n \quad u_1 = 2 \quad u_0 = 1 \quad (u_n)_{n \in \mathbb{N}}$$

$$\cdot v_n = u_{n+1} - u_n \quad \mathbb{N} \quad n$$

$$\cdot n \quad v_n \quad v_0 \quad (v_n)_{n \in \mathbb{N}} \quad - (1)$$

$$\cdot (u_n)_{n \in \mathbb{N}} \quad n \quad u_n \quad \forall n \in \mathbb{N}^* : u_n = u_0 + \sum_{k=0}^{n-1} v_k \quad - (2)$$

**:05** •

$$\cdot \forall n \in \mathbb{N} : u_n = 2u_{n+1} + 2n + 3 \quad u_0 = 2 \quad (u_n)_{n \in \mathbb{N}}$$

$$\cdot S_n = u_0 + u_1 + \dots + u_n = \sum_{k=0}^n u_k \quad \mathbb{N} \quad n$$

$$v_n = u_n + bn - 1 \quad (v_n)_{n \in \mathbb{N}} \quad b \quad - (1)$$

$$\cdot v_0$$

	$\mathbb{N}$	$n$	$n$	$u_n$	$v_n$	-2
	$\left(\frac{S_n}{n^2}\right)_{n \in \mathbb{N}^*}$	$\mathbb{N}$	$n$	$n$	$S_n$	-3
						<b>:06</b> •
	$(3-u_n)_{n \in \mathbb{N}}$	$(1+u_n)_{n \in \mathbb{N}}$		$(u_n)_{n \in \mathbb{N}}$		-1
				$(a_n)_{n \in \mathbb{N}}$	$b_n = \frac{a_n}{1+a_n}$	$\mathbb{N}$ $n$ -2
	(2):	$(a_n)_{n \in \mathbb{N}} \Rightarrow$	$(b_n)_{n \in \mathbb{N}}$	(1):	$\forall n \in \mathbb{N} : 0 \leq b_n \leq 1$	
	(4):	$(b_n)_{n \in \mathbb{N}} \Rightarrow$	$(a_n)_{n \in \mathbb{N}}$	(3):	$(a_n)_{n \in \mathbb{N}} \Rightarrow$	$(b_n)_{n \in \mathbb{N}}$
						<b>:07</b> •
		$(b_n)_{n \in \mathbb{N}^*}$	$(a_n)_{n \in \mathbb{N}^*}$			
	$\forall n \in \mathbb{N}^* : b_{n+1} = \frac{b_n}{n}$	$b_1 > 0$	$\forall n \in \mathbb{N}^* : a_{n+1} = \frac{a_n}{n}$	$a_1 < 0$		
		$(b_n)_{n \in \mathbb{N}^*}$	$(a_n)_{n \in \mathbb{N}^*}$	$\forall n \in \mathbb{N}^* : a_n < 0 < b_n$		-1
		$(b_n)_{n \in \mathbb{N}^*}$	$(a_n)_{n \in \mathbb{N}^*}$	$\forall n \in \mathbb{N}^* : b_n - a_n = \frac{b_1 - a_1}{(n-1)!}$		-2
						<b>:08</b> •
				$(v_n)_{n \in \mathbb{N}}$	$(u_n)_{n \in \mathbb{N}}$	
	$u_{n+1} = \frac{u_n + v_n}{2}$	$v_n = \frac{7}{u_n}$	$\mathbb{N}$	$n$	$u_0 = 3$	
	$\forall n \in \mathbb{N}^* : u_n - v_n = \frac{1}{4u_n} (u_{n-1} - v_{n-1})^2$			$(v_n)_{n \in \mathbb{N}}$	$(u_n)_{n \in \mathbb{N}}$	-1
		$\forall n \in \mathbb{N} : u_n - v_n \geq 0$				
		$\forall n \in \mathbb{N}^* : u_n \geq \frac{21}{8}$		$(v_n)_{n \in \mathbb{N}}$	$(u_n)_{n \in \mathbb{N}}$	-2
	$\forall n \in \mathbb{N} : 0 \leq u_n - v_n \leq \frac{1}{10^{2^n - 1}}$			$\forall n \in \mathbb{N}^* : u_n - v_n \leq \frac{1}{10} (u_{n-1} - v_{n-1})^2$		-3
				$(v_n)_{n \in \mathbb{N}}$	$(u_n)_{n \in \mathbb{N}}$	-4
						<b>:09</b> •
		$(v_n)_{n \in \mathbb{N}}$	$(u_n)_{n \in \mathbb{N}}$			
	$v_{n+1} = \frac{u_{n+1} + v_n}{2}$	$u_{n+1} = \frac{u_n + v_n}{2}$	$\mathbb{N}$	$n$	$v_0 = -1$	$u_0 = 2$
				$x_n = u_n - v_n$	$\mathbb{N}$	$n$ -1
	$n$	$x_n$	$x_0$	$q$	$(x_n)_{n \in \mathbb{N}}$	-1
				$(x_n)_{n \in \mathbb{N}}$		-ب
				$(v_n)_{n \in \mathbb{N}}$	$(u_n)_{n \in \mathbb{N}}$	-2
				$y_n = u_n + 2v_n$	$\mathbb{N}$	$n$ -3
	$\mathbb{N}$	$n$	$n$	$v_n$	$u_n$	$\forall n \in \mathbb{N} : y_n = 0$

$$\cdot (v_n)_{n \in \mathbb{N}} \quad (u_n)_{n \in \mathbb{N}} \quad \text{-(4)}$$

$$\cdot \lim_{n \rightarrow +\infty} S_n : \quad S_n = \sum_{k=0}^n u_k \quad n \quad \text{-(5)}$$

**:10** •

لكل  $a$  و  $b$  من  $\mathbb{R}$  بحيث  $0 < a < b$  نعتبر المتتاليتين  $(a_n)_{n \in \mathbb{N}}$  و  $(b_n)_{n \in \mathbb{N}}$  بحيث  $a_0 = a$  و  $b_0 = b$

$$\cdot \alpha_n = b_n - a_n \quad b_{n+1} = \frac{a_n + b_n}{2} \quad \text{و} \quad a_{n+1} = \frac{2a_n b_n}{a_n + b_n} : \mathbb{N} \quad n \quad \text{و}$$

$$\cdot \forall n \in \mathbb{N} : 0 \leq \alpha_n \leq \frac{b-a}{2^n} : \quad \forall n \in \mathbb{N} : 0 \leq \alpha_{n+1} \leq \frac{1}{2} \alpha_n : \quad \text{-(1)}$$

$$\cdot (b_n)_{n \in \mathbb{N}} \quad \text{و} \quad (a_n)_{n \in \mathbb{N}} \quad \text{متحاديتان} \quad \text{-(2)}$$

$$\cdot (b_n)_{n \in \mathbb{N}} \quad \text{و} \quad (a_n)_{n \in \mathbb{N}} \quad \forall n \in \mathbb{N} : a_n b_n = ab : \quad \text{-(3)}$$

**:11** •

$$f(x) = \frac{2x+1}{x+1} : \quad \mathbb{R} - \{-1\} \quad f$$

$$\cdot \begin{cases} v_0 = 2 \\ v_{n+1} = f(v_n); n \geq 0 \end{cases} \quad \begin{cases} u_0 = 1 \\ u_{n+1} = f(u_n); n \geq 0 \end{cases} : \quad (v_n)_{n \in \mathbb{N}} \quad (u_n)_{n \in \mathbb{N}}$$

$$\cdot f(I) \subseteq I : \quad I = [1, 2] \quad f \quad \text{-(1)}$$

$$\cdot (v_n)_{n \in \mathbb{N}} \quad (u_n)_{n \in \mathbb{N}} \quad (v_n)_{n \in \mathbb{N}} \subset I \quad (u_n)_{n \in \mathbb{N}} \subset I : \quad \text{-(2)}$$

$$\cdot \forall n \in \mathbb{N} : \begin{cases} v_n - u_n \geq 0 \\ v_{n+1} - u_{n+1} \leq \frac{1}{4}(v_n - u_n) \end{cases} : \quad \forall n \in \mathbb{N} : v_{n+1} - u_{n+1} = \frac{v_n - u_n}{(1+v_n)(1+u_n)} : \quad \text{-(3)}$$

$$\cdot (v_n)_{n \in \mathbb{N}} \quad (u_n)_{n \in \mathbb{N}} \quad \forall n \in \mathbb{N} : v_n - u_n \leq \frac{1}{4^n} : \quad \text{-(4)}$$

**:12** •

$$\cdot \forall n \in \mathbb{N} : u_{n+1} = -1 + \frac{1+u_n}{\sqrt{1+u_n^2}} \quad u_0 = -\frac{1}{2} : \quad (u_n)_{n \in \mathbb{N}}$$

$$\cdot f(I) \subseteq I \quad I = ]-1, 0[ \quad f : x \mapsto -1 + \frac{1+x}{\sqrt{1+x^2}} \quad \text{-(1)}$$

$$\cdot (u_n)_{n \in \mathbb{N}} \quad \forall n \in \mathbb{N} : u_n \in I : \quad \text{-(2)}$$

$$\cdot (u_n)_{n \in \mathbb{N}} \quad \text{-(3)}$$

**:13** •

$$\cdot \forall n \in \mathbb{N} : u_{n+1} = \sqrt{2-u_n} \quad u_0 = 2 : \quad (u_n)_{n \in \mathbb{N}}$$

$$\cdot (u_n)_{n \in \mathbb{N}} \quad f : x \mapsto \sqrt{2-x} : \quad \text{-(1)}$$

$$\cdot \forall n \in \mathbb{N} : |u_{n+1} - 1| \leq \frac{|u_n - 1|}{1 + \sqrt{2 - \sqrt{2}}} : \quad \forall n \in \mathbb{N} : 0 \leq u_n \leq \sqrt{2} : \quad \text{-(2)}$$

$$\cdot (u_n)_{n \in \mathbb{N}} \quad \forall n \in \mathbb{N}^* : |u_n - 1| \leq \frac{\sqrt{2} - 1}{(1 + \sqrt{2 - \sqrt{2}})^{n-1}} : \quad \text{-(3)}$$